⟨Quantum|Gravity⟩Society

Hidden Structures in Gravitational Scattering

Clifford Cheung

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Caltech

gravitational scattering: (1708.03872, 1705.03025, 2002.10470)
gravitational actions: (1612.00868, 1612.03927, 1709.04932, 2108.02276, 2204.07130)
applications to GWs: (1808.02489, 1901.04424, 1908.01493, 2003.08351, 2006.06665)

action

↓

amplitudes

“ *the theory* ”

action

↓

amplitudes

“ *the observables* ”

principles: *unitarity, locality, Poincare*

luxuries: *supersymmetry, extra dimensions, etc.*

action

amplitudes

action

↑ *"S-matrix program"*

amplitudes

Gauge symmetry manifests Poincare invariance and locality at the cost of redundancy.

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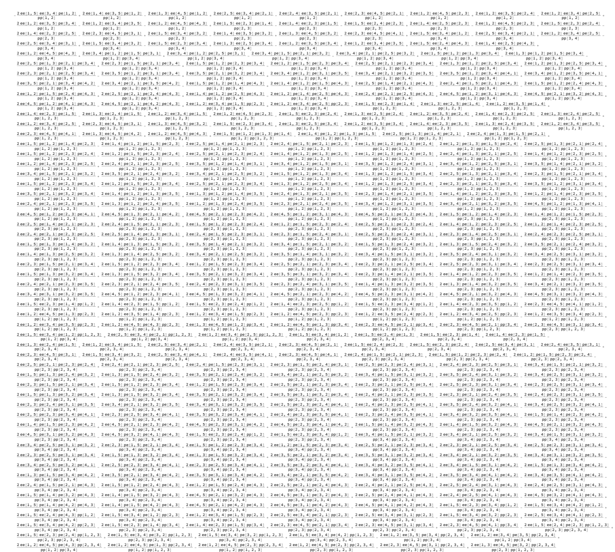
$$A(1^{h_1}2^{h_2}3^{h_3}4^{h_4}5^{h_5}) \quad = \quad$$



Feynman diagrams

(*factorization manifest*)

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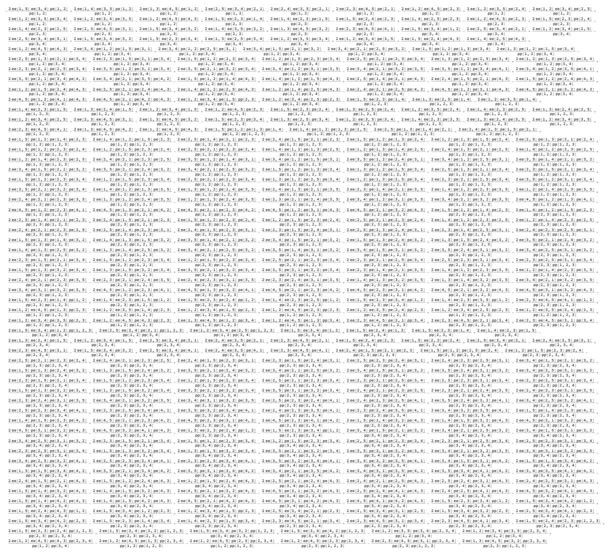

Feynman diagrams
(*factorization manifest*)

$$A(1^+2^+3^+4^+5^+) \quad = \quad A(1^-2^+3^+4^+5^+) \quad = 0$$

$$A(1^-2^+3^-4^+5^+) \quad = \quad \frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$$A(1^-2^-3^+4^+5^+) \quad = \quad \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

modern tools
(*factorization obscure*)

Gravity suffers also, due to diffeomorphisms.

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\&\qquad\qquad\qquad\qquad\qquad\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],\end{aligned} \tag{2.6}$$

3pt graviton *vertex*

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4pt graviton *vertex*

Gravity suffers also, due to diffeomorphisms.

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4pt graviton *vertex*

$$M(1^-2^-3^+) \quad = \quad \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

3pt graviton *amplitude*

$$M(1^-2^-3^+4^+) \quad = \quad \frac{\langle 12\rangle^4[34]^4}{stu}$$

4pt graviton *amplitude*

Redundancy is not an affliction of spin. Not even scalars are safe. Consider on-shell amplitudes in

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 g(\phi)$$

At 3pt, 4pt, 5pt, ... you will find they are all zero!

Redundancy is not an affliction of spin. Not even scalars are safe. Consider on-shell amplitudes in

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 g(\phi) \quad \longleftrightarrow \quad \mathscr{L} = \frac{1}{2}(\partial\phi)^2$$

At 3pt, 4pt, 5pt, ... you will find they are all zero!

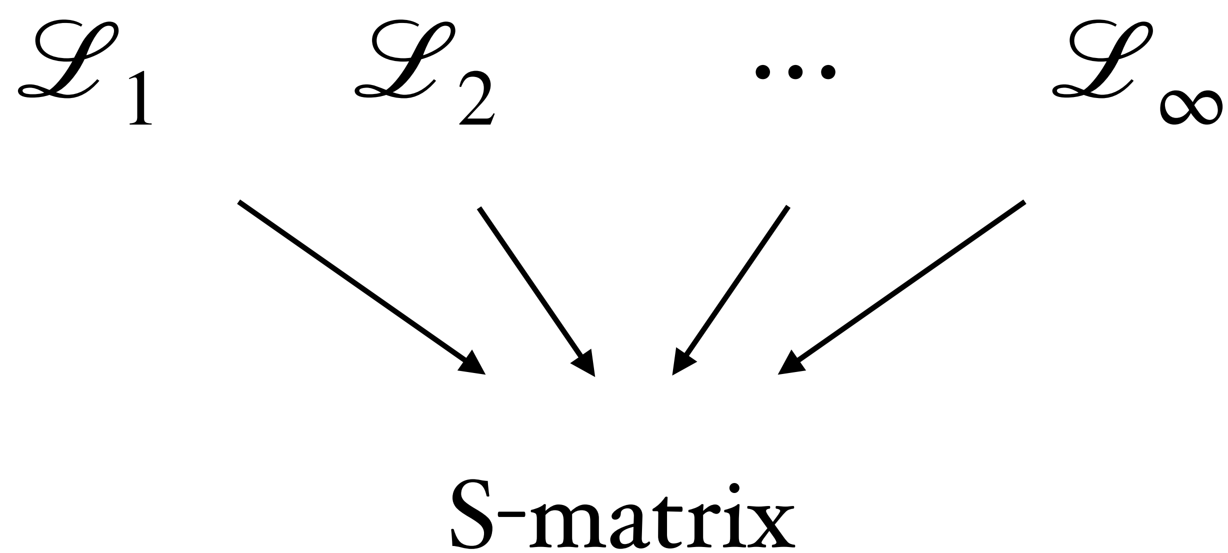
$$f(\phi) \longleftrightarrow \phi \qquad \text{where} \qquad f'(\phi)^2 = g(\phi)$$

Field redefinitions: a non-symmetry of the action that leaves the S-matrix invariant.

The fields of QFT are integration variables of the path integral. You can always change variables.

$$Z[J] \sim \int [d\phi]\, e^{iS[\phi]+i\int J\phi}$$

Thus, Lagrangians are infinitely redundant!



hidden structure #1: double copy

Bern, Carrasco, and Johansson (BCJ) discovered a hidden duality structure in gauge theory + gravity.

$$(\text{gauge})^2 = \text{gravity}$$

Color - Kinematics Duality: scattering exhibits an isomorphism between color and kinematics.

Double Copy: swapping color for kinematics yields the correct amplitudes of new theories.

In three-particle scattering, double copy is trivial.

3pt gluon

3pt graviton

$$A(1_a^- 2_b^- 3_c^+) = \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle} f_{abc} \qquad M(1^- 2^- 3^+) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

$$A(1_a^+ 2_b^+ 3_c^-) = \frac{[12]^3}{[13][32]} f_{abc} \qquad M(1^+ 2^+ 3^-) = \frac{[12]^6}{[13]^2[32]^2}$$

Simply replace f_{abc} with the kinematic structure.

In four-particle scattering, we see a small miracle.

$$A_4 = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

$$c_s = f_{abe} f_{cde} \qquad c_t = f_{bce} f_{ade} \qquad c_u = f_{cae} f_{bde}$$

Here n_s, n_t, n_u are non-unique functions of $p_i p_j$, $p_i e_j$, $e_i e_j$ that satisfy kinematic Jacobi identities.

$$c_s + c_t + c_u = 0 \qquad n_s + n_t + n_u = 0$$

(mathematical identity) (true on-shell)

Perhaps color and kinematics are interchangeable since they satisfy the same algebraic identities?

Bern, Carrasco, Johansson (0805.3993)

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4pt gluon
(*polarization* $= e_\mu$)

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4pt gluon
(*polarization* = e_μ)

$\downarrow\downarrow\downarrow$ " *double copy* "

$$M_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

4pt graviton
(*polarization* = $e_\mu e_\nu$)

Bern, Carrasco, Johansson (0805.3993)

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Double copy is proven at tree + recycled to loop integrands via unitarity to SUGRA, and LIGO.

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$$M_4 = \frac{n_s \tilde{n}_s}{s} + \frac{n_t \tilde{n}_t}{t} + \frac{n_u \tilde{n}_u}{u}$$

4pt graviton +
two-form + dilaton
(*polarization* $= e_\mu \tilde{e}_{\tilde{\mu}}$)

Double copy is proven at tree + recycled to loop integrands via unitarity to SUGRA, and LIGO.

Bern, Carrasco, Johansson (0805.3993)

Double copy is weirdly ubiquitous among “nice” theories with very few coupling constants.

$\mathcal{N} > 4$ supergravity	• $\mathcal{N} = 4$ SYM theory • SYM theory ($\mathcal{N} = 1, 2, 4$)	[1, 2, 31, 291, 292]	
$\mathcal{N} = 4$ supergravity with vector multiplets	• $\mathcal{N} = 4$ SYM theory • YM-scalar theory from dim. reduction	[1, 2, 31, 293]	• $\mathcal{N} = 2 \times \mathcal{N} = 2$ construction is also possible
pure $\mathcal{N} < 4$ supergravity	• (S)YM theory with matter • (S)YM theory with ghosts	[188]	• ghost fields in fundamental rep
Einstein gravity	• YM theory with matter • YM theory with ghosts	[188]	• ghost/matter fields in fundamental rep
$\mathcal{N} = 2$ Maxwell-Einstein supergravities (generic family)	• $\mathcal{N} = 2$ SYM theory • YM-scalar theory from dim. reduction	[120]	• truncations to $\mathcal{N} = 1, 0$ • only adjoint fields
$\mathcal{N} = 2$ Maxwell-Einstein supergravities (homogeneous theories)	• $\mathcal{N} = 2$ SYM theory with half hypermultiplet • YM-scalar theory from dim. reduction with matter fermions	[121, 294]	• fields in pseudo-real reps • include Magical Supergravities
$\mathcal{N} = 2$ supergravities with hypermultiplets	• $\mathcal{N} = 2$ SYM theory with half hypermultiplet • YM-scalar theory from dim. red. with extra matter scalars	[121, 240]	• fields in matter representations • construction known in particular cases
$\mathcal{N} = 2$ supergravities with vector/ hypermultiplets	• $\mathcal{N} = 1$ SYM theory with chiral multiplets • $\mathcal{N} = 1$ SYM theory with chiral multiplets	[239, 241, 295]	• construction known in particular cases
$\mathcal{N} = 1$ supergravities with vector multiplets	• $\mathcal{N} = 1$ SYM theory with chiral multiplets • YM-scalar theory with fermions	[188, 239, 241, 295]	• fields in matter reps • construction known in particular cases
$\mathcal{N} = 1$ supergravities with chiral multiplets	• $\mathcal{N} = 1$ SYM theory with chiral multiplets • YM-scalar with extra matter scalars	[188, 239, 241, 295]	• fields in matter reps • construction known in particular cases
Einstein gravity with matter	• YM theory with matter • YM theory with matter	[1, 188]	• construction known in particular cases
$R + \phi R^2 + R^3$ gravity	• YM theory + F^3 + F^4 + ... • YM theory + F^3 + F^4 + ...	[296]	• extension to $\mathcal{N} \leq 4$ replacing one of the factors by undeformed SYM theory
Conformal (super)gravity	• DF^2 theory • (S)YM theory	[152, 153]	• $\mathcal{N} \leq 4$ • involves specific gauge theory with dimension-six operators
$3D$ maximal supergravity	• BLG theory • BLG theory	[119, 243, 297]	• $3D$ only
YME supergravities	• SYM theory • YM + ϕ^3 theory	[120, 125, 133, 134, 140, 214, 216, 257, 283, 285, 289]	• trilinear scalar couplings • $\mathcal{N} = 0, 1, 2, 4$ possible
Higgsed supergravities	• SYM theory (Coulomb branch) • YM + ϕ^3 theory with extra massive scalars	[122]	• $\mathcal{N} = 0, 1, 2, 4$ possible • massive fields in supergravity
$U(1)_R$ gauged supergravities	• SYM theory (Coulomb branch) • YM theory with SUSY broken by fermion masses	[123]	• $0 \leq \mathcal{N} \leq 8$ possible • SUSY is spontaneously broken • only theories with Minkowski vacua
gauged supergravities (nonabelian)	• SYM theory (Coulomb branch) • YM + ϕ^3 theory with massive fermions	[284]	• SUSY is spontaneously broken • only theories with Minkowski vacua
DBI theory	• NLSM • (S)YM theory	[125, 126, 285, 298–301]	• $\mathcal{N} \leq 4$ possible • also obtained as $\alpha' \to 0$ limit of abelian Z-theory
Volkov-Akulov theory	• NLSM • SYM theory (external fermions)	[125, 302–308]	• restriction to external fermions from supersymmetric DBI
Special Galileon theory	• NLSM • NLSM	[125, 285, 301, 306, 309]	• theory is also characterized by its soft limits
DBI + (S)YM theory	• NLSM + ϕ^3 • (S)YM theory	[125, 126, 156, 285, 298–300, 306, 310]	• $\mathcal{N} \leq 4$ possible • also obtained as $\alpha' \to 0$ limit of semi-abelianized Z-theory
DBI + NLSM theory	• NLSM • YM + ϕ^3 theory	[125, 126, 156, 285, 298–300]	

Bern, Carrasco, Chiodaroli, Johansson, Roiban (1909.01358)

Double copy transcends gauge theory + gravity!

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Is the double copy just the tetrad formalism? No.

Double copy transcends gauge theory + gravity!

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- gluon $\otimes$ pion = Born-Infeld photon

Is the double copy just the tetrad formalism? No.
Is it just open/closed string duality? Unclear.
Anyway, a QFT fact deserves a QFT explanation.

The double copy is a proven fact about on-shell, flat-space, tree-level scattering amplitudes.

Why is it true?

The double copy is a proven fact about on-shell, flat-space, tree-level scattering amplitudes.

Why is it true? When is it true?

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Why is it true? When is it true?

- which theories?
- higher-loops?
- non-perturbatively?
- curved spacetime?
- classical solutions?

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We don't understand double copy. And the stakes are not low: (lattice QCD) ^{2} = QG ?

Case in point, consider double copy amplitude for the theory of the graviton + two-form + dilaton.

$$M_4 = \frac{n_s \tilde{n}_s}{s} + \frac{n_t \tilde{n}_t}{t} + \frac{n_u \tilde{n}_u}{u}$$

$$n_s, n_t, n_u \;=\; \text{functions of } e_\mu, p_\mu$$

$$\tilde{n}_s, \tilde{n}_t, \tilde{n}_u \;=\; \text{functions of } \tilde{e}_{\tilde{\mu}}, p_{\tilde{\mu}}$$

So there is an amplitudes representation with two Lorentz invariances acting on μ and $\tilde{\mu}$ indices.

Can the double Lorentz invariance of gravity be made explicit in the off-shell action?

$$S_{\rm EH} = \int d^d x \sqrt{-g} \left(\frac{R}{16\pi G} + L_{\rm GF} \right)$$

arbitrary

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) + O(h^3) + \cdots$$

We can exploit the freedom of field basis and gauge fixing which leaves amplitudes unchanged.

CC, Remmen (1612.03927)

The resulting action is remarkably compact.

$$S_{\rm EH} = \frac{1}{16\pi G}\int d^d x\, \partial_A \Sigma_{CE} \partial_B \Sigma^{DE} \left(\frac{1}{16}\Sigma^{AB}\delta^C_D - \frac{1}{4}\Sigma^{CB}\delta^A_D \right)$$

$$\Sigma_{AB} = (e^H)_{AB}$$

$$\Sigma^{AB} = (e^{-H})^{AB}$$

$$H_{AB} = \overset{\quad d \;\; + \;\; d}{\begin{pmatrix} 0 & h_{\mu\tilde{\nu}} \\ h_{\tilde{\mu}\nu} & 0 \end{pmatrix}}$$

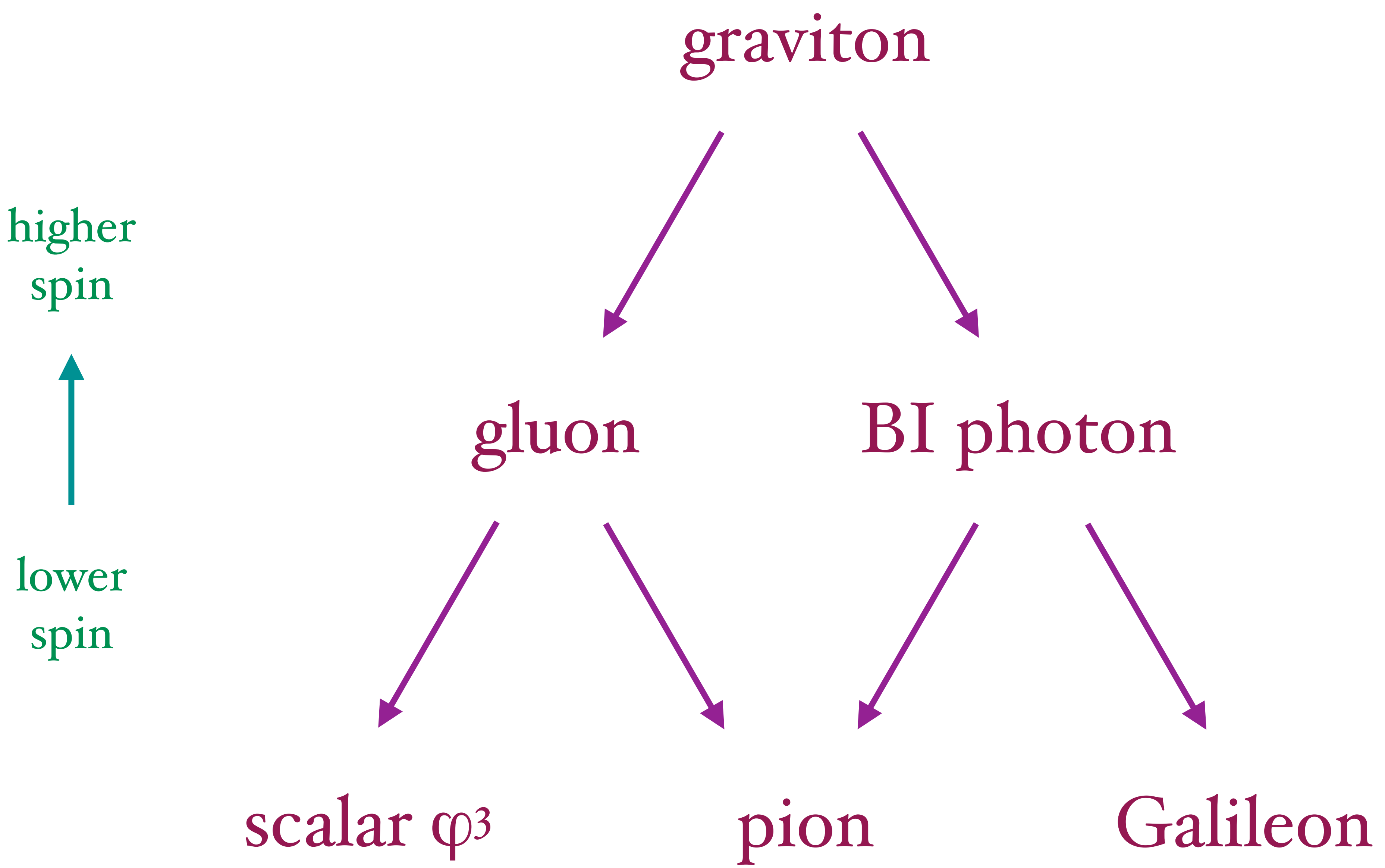
$$\partial_A = \overset{\quad d \;\; + \;\; d}{(\,\partial_\mu\,,\,\partial_{\tilde{\mu}}\,)}$$

The off-shell Feynman diagrams have a doubled Lorentz invariance and the on-shell amplitudes have a doubled gauge symmetry.

CC, Remmen (1612.03927)

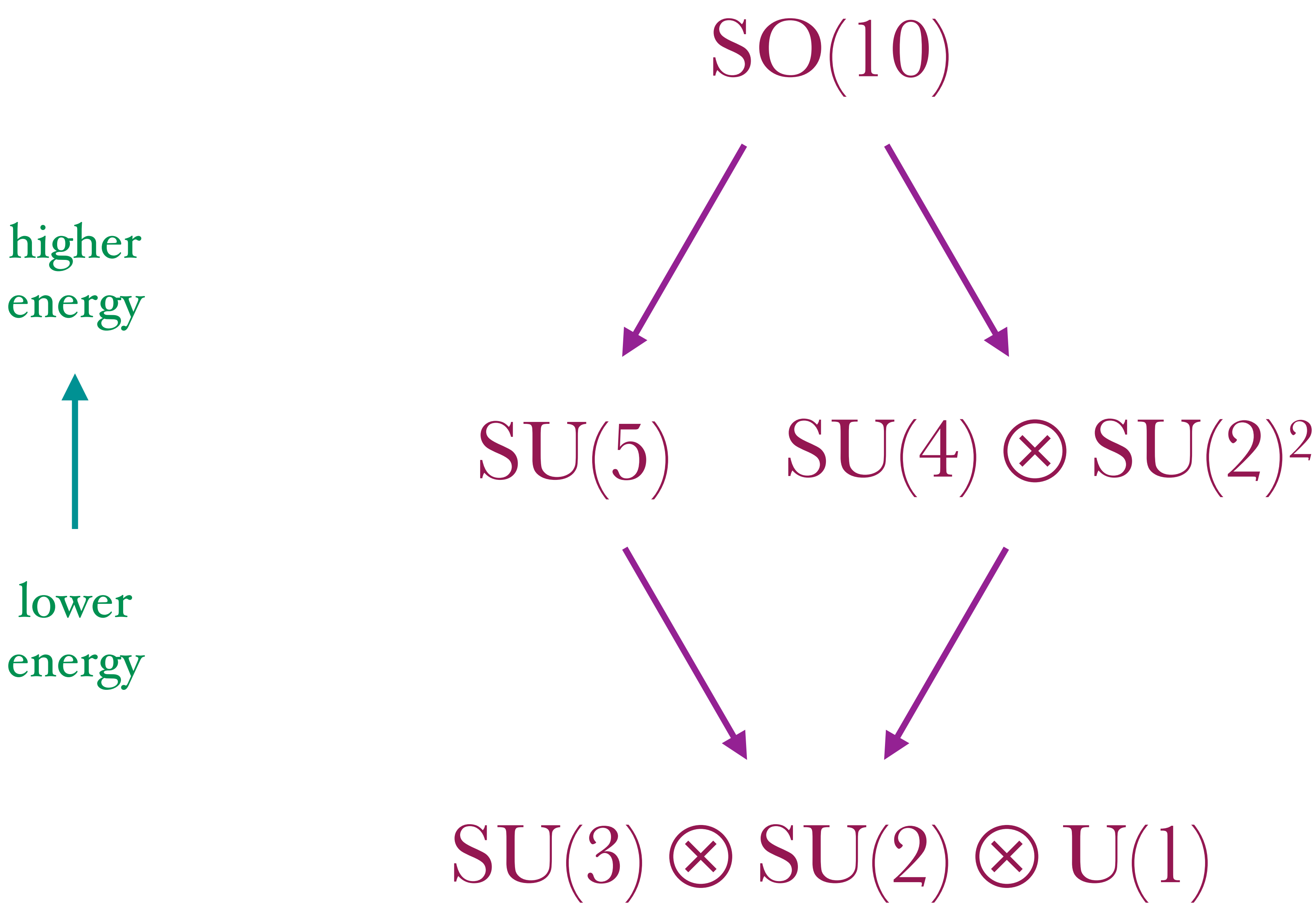
hidden structure #2: unification via gravity

The graviton S-matrix encodes gluons, pions, etc.



CC, Shen, Wen (1705.03025)

This is distinct from textbook grand unification.



To begin, we think of gluon tree amplitudes as abstract functions of kinematic invariants.

$$A = e_1^{\mu_1} e_2^{\mu_2} \cdots e_n^{\mu_n} A_{\mu_1 \mu_2 \cdots \mu_n}$$

$$= \text{scalar function of } p_i p_j, p_i e_j, e_i e_j$$

Crucially, we maintain the on-shell conditions.

massless helicity basis

$$p_i p_i = p_i e_i = e_i e_i = 0$$

transverse

A physical on-shell amplitude satisfies several constraints. The first is the Ward identity.

$$A\Big|_{e_i=p_i} \quad = \quad W_i\, A \quad = \quad 0$$

Here we recast the Ward identity as a differential operator that annihilates the amplitude.

$$W_i \quad = \quad \sum_{v=p_j,\, e_j} (p_i v) \frac{\partial}{\partial\,(v e_i)}$$

The second constraint is typically trivial: total momentum conservation.

$$P_v A = 0$$

As before, we can define an operator for this property of the amplitudes.

$$P_v = \sum_i p_i v$$

Now let us construct an operator T that acts on the amplitude A to produce a new one $T \cdot A$.

If the operator satisfies the conditions,

$$[\, W_i \,, T \,] \sim 0 \qquad\qquad [\, P_\nu \,, T \,] \sim 0$$

then if A is gauge invariant and momentum-conserving then so too is $T \cdot A$.

$$W_i \cdot (\, T \cdot A \,) = 0 \qquad\qquad P_\nu \cdot (\, T \cdot A \,) = 0$$

From these vanishing commutators we derive the "transmutation operators".

$$T_{ij} = \frac{\partial}{\partial (e_i e_j)}$$

2 gluon → 2 scalar

$$T_{ijk} = \frac{\partial}{\partial (p_i e_j)} - \frac{\partial}{\partial (p_k e_j)}$$

1 gluon → 1 scalar

$$T_i = \sum_j p_i p_j \frac{\partial}{\partial (p_j e_i)}$$

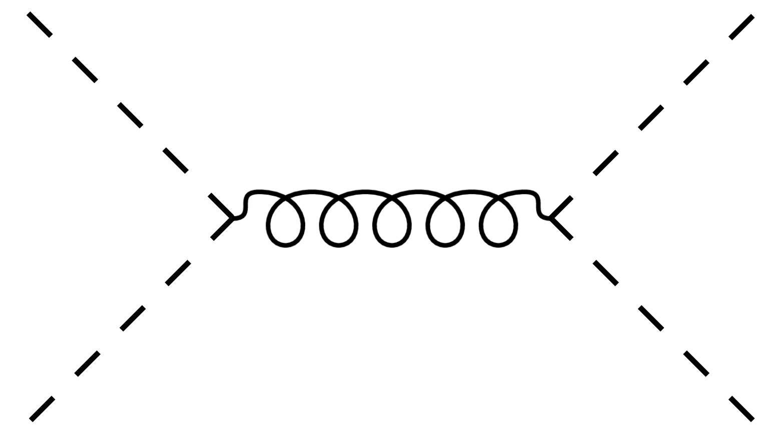
1 gluon → 1 pion

We proved transmutation for all graviton, gluon, pion tree amplitudes + explicit checks up to 8pt.

Example #1: YM to SQED

$$T_{12} \cdot T_{34} \cdot A(g_1, g_2, g_3, g_4) = \left[\frac{\partial}{\partial (e_1 e_2)} \frac{\partial}{\partial (e_3 e_4)} \right] A(g_1, g_2, g_3, g_4)$$

$$= \frac{p_1 p_3}{p_1 p_2} = A(\phi_1, \phi_2, \phi_3, \phi_4) =$$



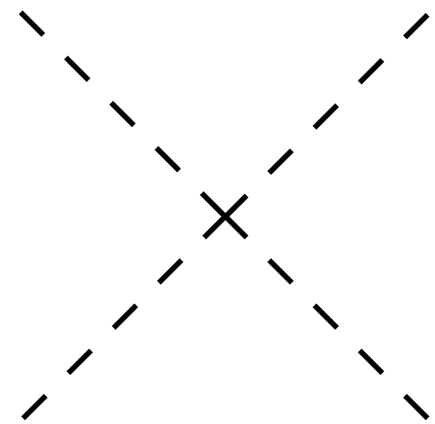
Extracting the $(e_1 e_2)(e_3 e_4)$ term is dimensional reduction to two new flavors of charged scalars.

$$e_1^\mu = e_2^\mu = \overset{d+1+1}{(0,1,0)} \qquad e_3^\mu = e_4^\mu = \overset{d+1+1}{(0,0,1)}$$

Example #2: YM to NLSM

$$T_{14} \cdot T_2 \cdot T_3 \cdot A(g_1, g_2, g_3, g_4) = \left[\frac{\partial}{\partial\,(e_1 e_4)} \cdots \right] A(g_1, g_2, g_3, g_4)$$

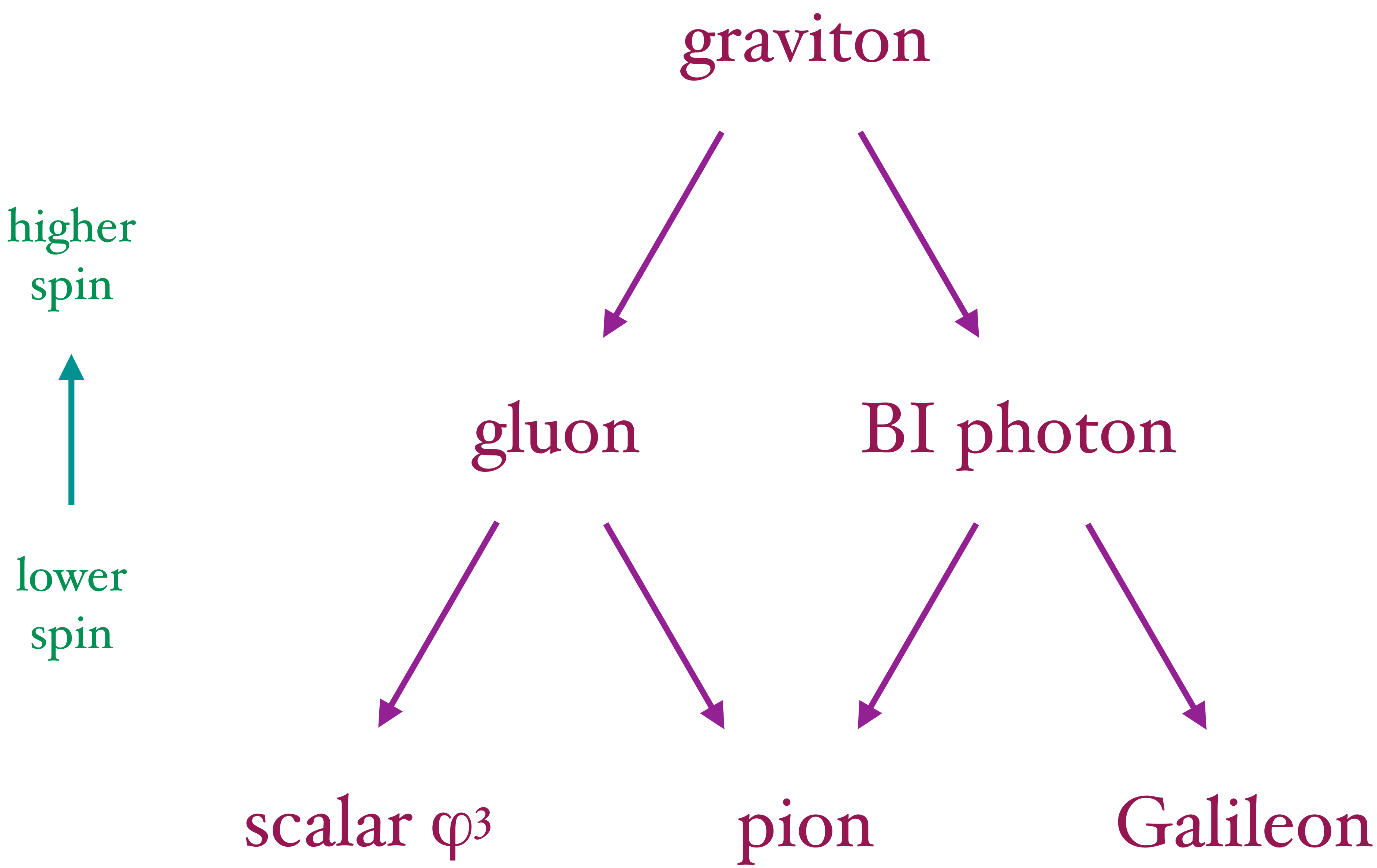
$$= p_1 p_3 = A(\pi_1, \pi_2, \pi_3, \pi_4) =$$



We can recast pions as oddly polarized gluons under dimensional reduction.

$$e_1^\mu = e_2^\mu = \overbrace{(0,1,0)}^{d+1+d} \qquad e_2^\mu = \overbrace{(p_2^\alpha, 0, i p_2^\beta)}^{d+1+d} \qquad e_3^\mu = \overbrace{(p_3^\alpha, 0, i p_3^\beta)}^{d+1+d}$$

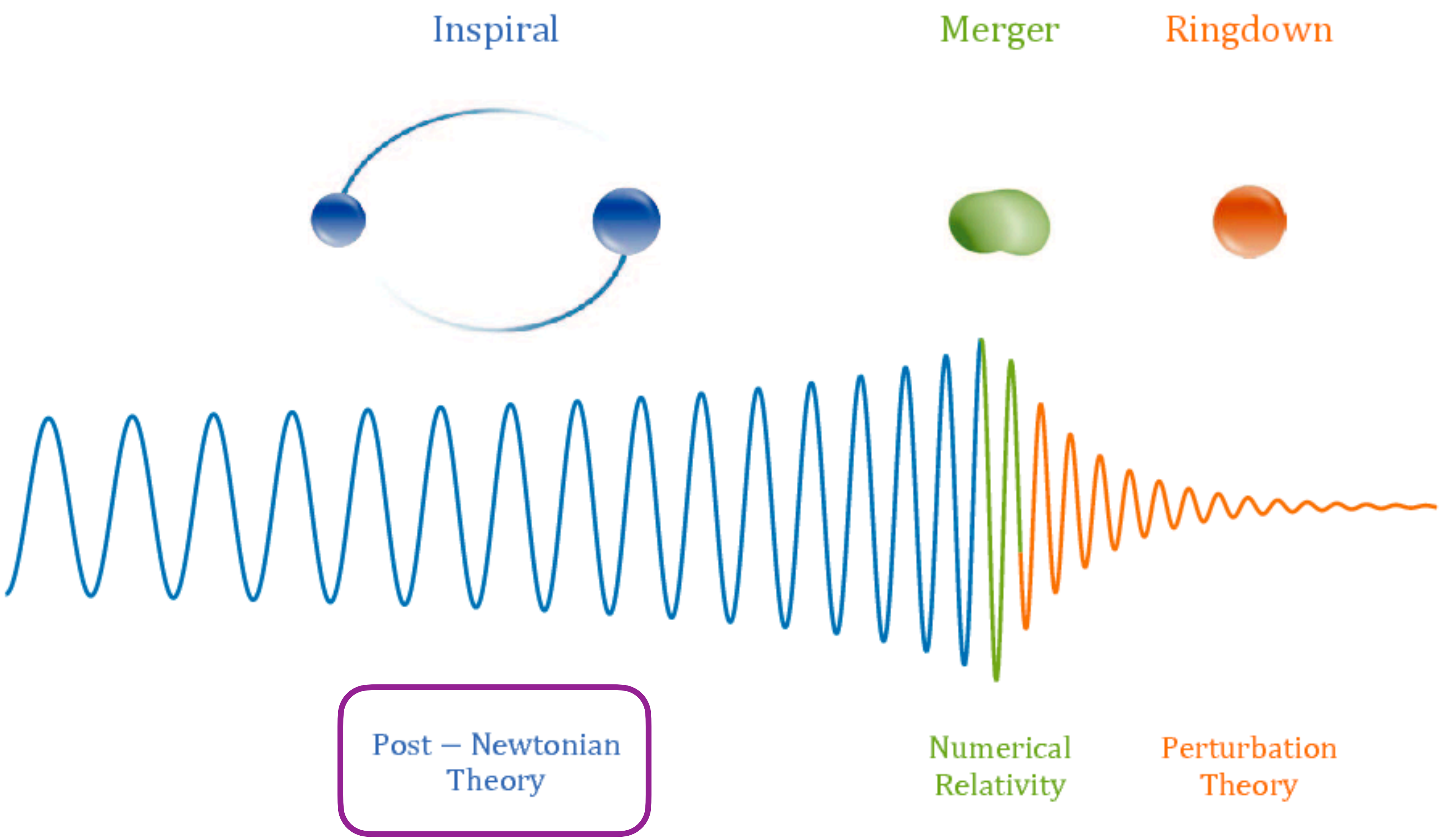
“ gravity = mother of all theories ”



CC, Shen, Wen (1705.03025)

applications to gravitational waves

The binary black hole merger has three phases.



perturbation theory
is applicable here

figure from 1610.03567

State-of-the-art perturbative computations in gravitational wave physics center on the "post-Newtonian" expansion, based on

$$v^2 \sim \frac{GM}{r} \ll 1$$

which is tiny and perturbatively calculable during the inspired phase of the merger.

The so-called "post-Minkowskian" expansion parameter is G, and we call it perturbation theory.

Map of Perturbation Theory

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

$$\text{1PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots \right) G$$

$$\text{2PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots \right) G^2$$

$$\text{3PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots \right) G^3$$

$$\text{4PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + \cdots \right) G^4$$

$$\text{5PM} \quad \left(1 + v^2 + v^4 + v^6 + \cdots \right) G^5$$

$$\vdots$$

Map of Perturbation Theory

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$\big(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ v^{14}$	$+ \cdots \big) G$
2PM		$\big(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ \cdots \big) G^2$
3PM			$\big(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ \cdots \big) G^3$
4PM				$\big(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ \cdots \big) G^4$
5PM					$\big(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ \cdots \big) G^5$
						$\vdots$			

Can amplitudes give an efficient and scaleable path to higher PN? Naively, there are issues.

- black holes $\neq$ SYM gluons

  (double copy, recursion, etc. all apply to masses)

- LIGO does not observe scattering

  (NRQCD solved the amplitudes - potentials map)

All these puzzles have been surmounted. New results on conservative dynamics, radiation, spin, finite size effects are appearing swiftly.

full theory		effective theory
↓ amplitudes methods		↓ build ansatz
BH / graviton tree amplitudes: A_{tree}		$V(p,q)$: effective BH Lagrangian
↓ generalized unitarity		↓ Feynman diagrams
integral representation: $A = \sum_i d^{(i)} I^{(i)}$		$A_{\text{EFT}} = \sum_i d^{(i)}_{\text{EFT}} I^{(i)}$: integral representation
↓ multi-loop integration		↓ multi-loop integration
full loop amplitude: $A(p,q)$	identical physics $=$	$A_{\text{EFT}}(p,q)$: EFT loop amplitude

gluons

(double copy) $\downarrow$ $$A_{\text{grav}} = A_{\text{YM}} \otimes A_{\text{YM}}$$

gravitons

(transmute) $\downarrow$ $$A_{\phi_1\phi_2+\text{grav}} = \frac{\partial A_{\text{grav}}}{\partial(e_1e_2)}$$

massless scalars + gravitons

(add mass) $\downarrow$ $$A_{\phi_1'\phi_2'+\text{grav}} = A_{\phi_1\phi_2+\text{grav}}\Big|_{p_1p_2 \to p_1p_2 - m_1m_2}$$

massive scalars + gravitons

Map of Perturbation Theory

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

1PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots\right) G$$

2PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots\right) G^2$$

3PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) G^3$$

4PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) G^4$$

5PM $$\left(1 + v^2 + v^4 + v^6 + \cdots\right) G^5$$

$$\vdots$$

Map of Perturbation Theory

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

1PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots\right) G$$

2PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots\right) G^2$$

3PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) G^3$$

4PM $$\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) G^4$$

5PM $$\left(1 + v^2 + v^4 + v^6 + \cdots\right) G^5$$

$$\vdots$$

1808.02489

1901.04424

These amplitudes methods are now state-of-the-art approach for the PM conservative potential.

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right),$$

$$c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\left[\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} + \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3}$$
$$\left. - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\right]$$

Bern, CC, Roiban, Shen, Solon, Zeng (1901.04424, 1908.01493)

conclusions

- Scattering amplitudes have uncovered hidden structures lurking inside real-world theories like gravitons, gluons, and pions.

- In my view, these structures should be visible as symmetries or principles in an action. Some progress has been manifesting enhanced Lorentz and color-kinematics symmetries.

- Double copy, generalized unitarity and EFT have together led to state-of-the-art results for gravitational wave physics, with more to come!

thank you!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