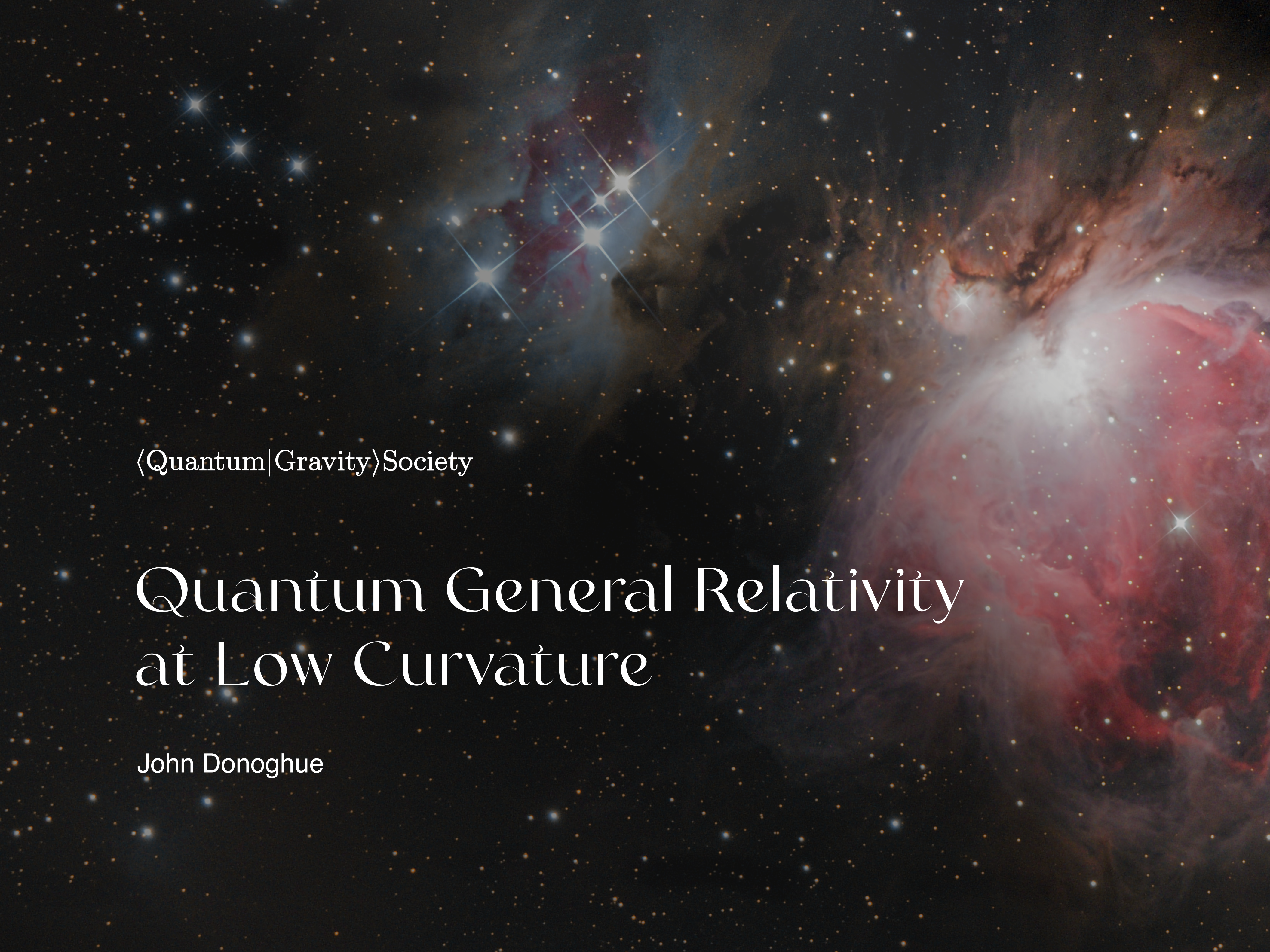


⟨Quantum|Gravity⟩Society

Quantum General Relativity at Low Curvature

John Donoghue

Quantum General Relativity at Low Curvature

Overview

1) QED vs EFT of General Relativity (QGR similar to low energy QED)

2) Some lessons of low energy quantum gravity

- Limits of the EFT

3) A novelty: The nonlocal partner of the cosmological constant

Amherst Center for Fundamental Interactions
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

John Donoghue
Vancouver
Aug 15, 2022

Some topics for hallways discussions

Ongoing work with
Gabriel Menezes

Quadratic gravity

- renormalizable QFT with metric as the primary DOF

Arrow of time

- QFT has an arrow of causality $\Rightarrow$ arrow of thermodynamics
- determined from the i in e^{iS}

Ostrogradsky

- classical vs quantum role of Hamiltonian in higher derivative theories
- in some cases, quantum stability

Logarithmic running in theories with intrinsic mass scale is not always real running

- Asymptotic Safety, Quadratic Gravity…

Our Core Theory

$$Z^{core} = \int [d\phi d\psi dA dg]_\Lambda \left[-\frac{1}{4}F^2 + \bar{\psi} i D\psi + \frac{1}{2}\partial\phi\partial\phi - V(\phi) - \Gamma\bar{\psi}\phi\psi - \Lambda_{cc} + \frac{2}{\kappa^2}R + ... \right]$$

YM and gravity most straightforward with Path Integrals

The cutoff Λ represents our incomplete knowledge of unexplored high energy realm

All of SM is expected to be an Effective Field Theory

Primacy of quantum theory – some understanding of obtaining classical physics

General Relativity fits naturally in this paradigm at present energies

1) Detour into QED

$$Z[J] = \int [dA_\mu d\psi]_\Lambda \; e^{i\int d^4x[\mathcal{L}_{QED}(A,\psi) - J_\mu A^\mu]}$$

Photon is low energy DOF

- not at all energies – hence $\Lambda \sim M_W$

At very low energies one can integrate out the massive fermion

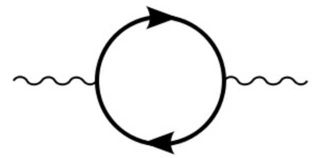
$$Z[J] = \int [dA_\mu]_m \; e^{i\int d^4x[\mathcal{L}_{eff}(A) - J_\mu A^\mu]}$$

with

$$e^{i\int d^4x \mathcal{L}_{eff}(A)} = \int [d\psi]_\Lambda e^{i\int d^4x[\mathcal{L}_{QED}(A,\psi)}$$

Matching

Match to perturbative calculations:



$$\hat{\Pi}_{\mu\nu}(q) = \frac{\alpha}{15\pi}\left(q_\mu q_\nu - \eta_{\mu\nu} q^2\right)\frac{q^2}{m^2} \qquad (q^2 << m^2)$$

$$= -\frac{\alpha}{3\pi}\left(q_\mu q_\nu - \eta_{\mu\nu} q^2\right)[\log\frac{-q^2}{m^2}] \qquad (q^2 >> m^2)$$

Can read off effective Lagrangian

$$\mathcal{L}_{eff}(A) = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{240\pi^2 m^2}F_{\mu\nu}\Box F^{\mu\nu}$$

Local Lagrangian

- uncertainty principle

Higher derivative Lagrangian but no Ostrogradsky instability

Massless fields

Non-local – long distance propagation

Integrating out the massless charged particle

$$\mathcal{L}_{eff}(A) = -\frac{1}{4e^2(\mu)}F_{\mu\nu}F^{\mu\nu} - \frac{1}{48\pi^2}F_{\mu\nu}\log\frac{\Box}{\mu^2}\ F^{\mu\nu}$$

Despite quasi-local notation, this is non-local

$$\int d^4x\ F_{\mu\nu}\log\frac{\Box}{\mu^2}\ F^{\mu\nu} = \int d^4x d^4y\ F_{\mu\nu}(y)\ \langle y|\log\frac{\Box}{\mu^2}|x\rangle\ F^{\mu\nu}(x)$$

with the function

$$\langle y|\log\Box|x\rangle = \int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}\log(-q^2)$$

Detour to our QED detour – covariant non-local actions

In Yang-Mills, fields transform differently at different positions

Wilson line can make a covariant action

$$\int d^4x \; F^A_{\mu\nu} \log\frac{\Box}{\mu^2} F^{A\mu\nu} = \int d^4x d^4y \; F^A_{\mu\nu}(y) \; W^{AB}(y-x) \; \langle y| \log\frac{\Box}{\mu^2} |x\rangle \; F^{B\mu\nu}(x)$$

where $$W^{AB}(y-x) = \left[P \int_x^y dz^\mu t^C A^C_\mu(z) \right]^{AB}$$

Verified in a next order calculation

JD and El-Menoufi

Another detour - adding classical gravity to the QED action

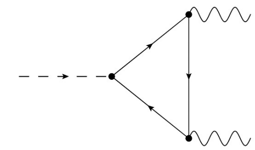


FIG. 1: Triangle diagram.

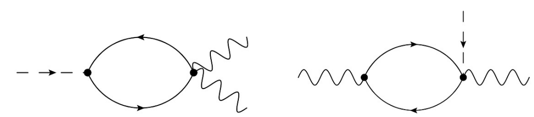


FIG. 2: Bubble diagrams.

Two photon-graviton matrix element has pole-like non-localities:

$$\mathcal{M}^f_{\mu\nu,\alpha\beta} = \frac{1}{24\pi^2 q^2}\left(-Q_\mu Q_\nu - q_\mu q_\nu + q^2\eta_{\mu\nu}\right)\left(p'_\alpha p_\beta - p\cdot p'\eta_{\alpha\beta}\right)$$

Yield covariant nonlocal action by matching

$$\overset{(3)}{\Gamma}_{pole}[g,A] = \int d^4x\sqrt{g}\big(\bar{P}^S F_{\mu\nu}F^{\mu\nu}\frac{1}{\nabla^2}R + P^C F_\alpha{}^\beta F^{\mu\nu}\frac{1}{\nabla^2}C^\alpha{}_{\beta\mu\nu}\big)$$

where

$$\bar{P}^S = -\frac{1}{576\pi^2}, \quad P^C = -\frac{1}{96\pi^2} \quad .$$

The EFT of general relativity

Gravitons are the quantum fields at ordinary energy

- don't need to know UV completion
- high energy gives local effects

Action will contain local terms consistent with general covariance

$$S = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + c_1(\mu) R^2 + c_2(\mu) R_{\alpha\beta} R^{\alpha\beta} + \ldots \right]$$

How to tell unique low energy effects – non-analyticity/ non-locality

$$\left[1 + a' G(M+m)\sqrt{-q^2} + b' G\hbar q^2 \ln(-q^2) + c' G q^2 \right]$$

2) Lessons of quantum General Relativity

We have a quantum theory of general relativity, valid is some domain

- what are some implications?

Warning – effects are small in domain of validity

- best perturbation theory ever

But trends are interesting

- GR is classical limit of some complete quantum theory
- EFT is an intermediate stage

Universality of the non-relativistic potential

Defined from the Fourier transform of non-relativistic scattering amplitude

- well defined, gauge invariant

Non-analyticities - $\sqrt{-q^2}$ and $\log -q^2$

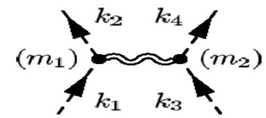
$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2\right]$$

Universal result – independent of spin ("soft theorem")

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

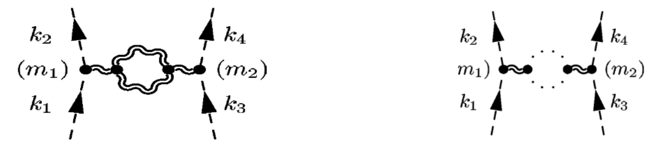
The calculation:

Lowest order:

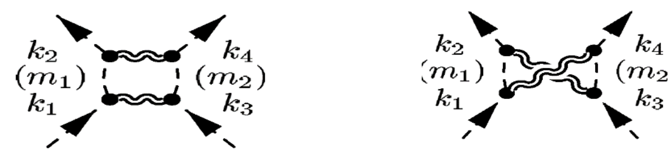


Vertex corrections:

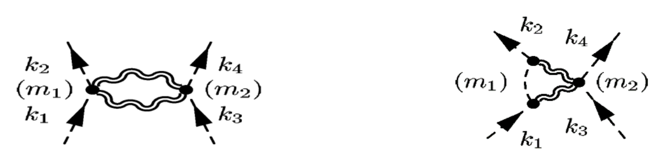
Vacuum polarization:
(Duff 1974)



Box and crossed box



Others:

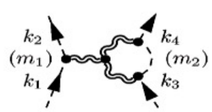


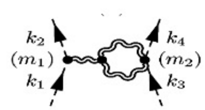
Classical and quantum in the same diagrams

Classical part from Feynman diagrams

This is the $\sqrt{-q^2}$ non-analyticity

Both $\sqrt{-q^2}$ and $\log -q^2$ arise Feynman diagrams





$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right)$$

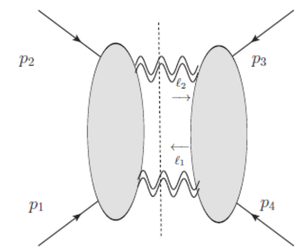
$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2$$

This has become an industry – Post-Newtonian/Minkowski expansion

On-shell amplitudes determine leading quantum effect

Modern unitarity based techniques

- basically $\log -q^2 = \log |q^2| - i\pi\theta(q^2)$ plus discontinuity rules
- no need for ghosts
- can use on-shell "gravity is square of gauge theory" amplitudes



And also dispersion relations – on shell spectral function

- independent of number of subtractions

$$V(s,q^2) = -\frac{1}{\pi}\int_0^\infty dt \frac{1}{t-q^2}\rho(s,t) + \text{R.H.cut}\,.$$

Bjerrum-Bohr
JFD
Vanhove

$$\rho(s,t) = -\frac{1}{\pi}\int \frac{d\Omega_\ell}{4\pi} M^{\text{tree}}_{\mu\nu,\rho\sigma}(p_1,p_2,-\ell_2,\ell_1)\mathcal{S}^{\mu\nu,\alpha\beta}\mathcal{S}^{\rho\sigma,\gamma\delta}(M^{\text{tree}}_{\alpha\beta,\gamma\delta}(p_4,p_3,\ell_2,-\ell_1))^*\,.$$

This explains universality

QED and gravity Compton amplitudes are universal Low, Weinberg Gross, Jackiw

– tree level soft theorems

- leads to loop-level soft theorems

Non-universality of bending of massless particles

Use peak of eikonal phase as definition of bending angle

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\text{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

But with different coefficients

$$\text{bu}^\eta = (371/120,\ 113/120,\ \ -29/8)$$

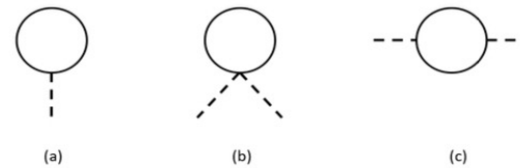
for massless scalar, photon, graviton

Reproduces first two terms in the classical expansion

Of course, also spread in quantum bending angles

G and Λ are not running parameters in physical reactions

Renormalization of cosmological constant



$$\sqrt{-g}\Lambda = \Lambda(1 + \frac{1}{2}h^\sigma_\sigma + \frac{1}{8}(h^\sigma_\sigma)^2 - \frac{1}{4}h_{\sigma\lambda}h^{\sigma\lambda} + ...)$$

- example here due to massive scalar

$$-i\mathcal{M} = i\frac{m^4}{32\pi^2}\left(\frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{\mu\nu}h_{\mu\nu}\right)\left[\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m^2} + \frac{3}{2}\right]$$

- no dependence on external momentum scales
- $\log \mu$ disappears along with $\frac{1}{\epsilon}$ upon renormalization

No useful and universal definition in physical processes

- Energy dependence is not universal – varies (a lot) between processes
- even changes sign
- survey of many reactions

There is no “quantum corrected metric”

The metric is a dynamical quantum field

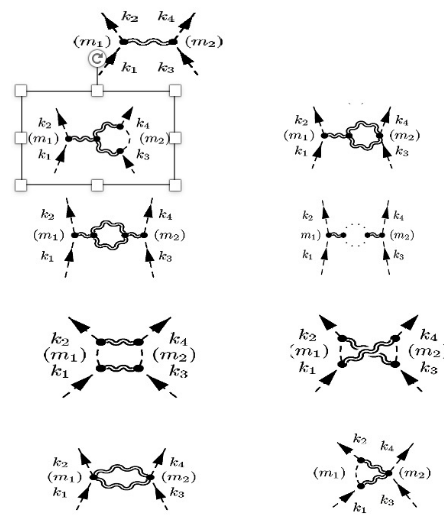
Can’t use geodesics in some “quantum metric”
- can be seen in direct calculations
- non-universality of quantum corrections

All diagrams enter at same level

Tidal effects cannot be removed

No test particle limit

Mea culpa



Lightcones are ill-defined

Massless fields do not follow identical trajectories

Penrose diagram is approximation of unknown uncertainty

Leads to global causal uncertainty

But at low energy this can be masked by usual uncertainty

Side note: Quadratic gravity has Planck scale causality violation

Nonlocal effective actions

Pioneered by Barvinsky and Vilkovisky

- nonlocal heat kernel or matching to perturbative calculation

For example, logarithmic corrections in gravitational sector

$$S = \int d^4x \sqrt{-g} \left[c_1(\mu) R^2 + c_2(\mu) R_{\mu\nu} R^{\mu\nu} + \alpha R \log \frac{\Box}{\mu^2} R + \beta R_{\mu\nu} \log \frac{\Box}{\mu^2} R^{\mu\nu} \right]$$

Pole corrections at higher order in the curvature

$\sim R^2 \frac{1}{\Box} R$ similar to $F_{\mu\nu} F^{\mu\nu} \frac{1}{\nabla^2} R +$

But these are at the **same order** in the derivative expansion

A fun quote:

"A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

Frank Wilczek
Physics Today
2002

3) A novelty – nonlocal partner to the cosmoligical constant

We have seen a connection of divergences and non-localities

$$S = \int d^4x \sqrt{-g} \left[c_1(\mu) R^2 + c_2(\mu) R_{\mu\nu} R^{\mu\nu} + \alpha R \log \frac{\Box}{\mu^2} R + \beta R_{\mu\nu} \log \frac{\Box}{\mu^2} R^{\mu\nu} \right]$$

Bubble and triangle diagrams

For a massive particle in a loop, we also expect other non-localities

- second order in derivative expansion $\frac{1}{\epsilon} m^2 \sqrt{-g} R + \text{ nonlocal}$

- zeroth order in derivative expansion $\frac{1}{\epsilon} m^4 \sqrt{-g} + \text{ nonlocal}$

What are these effects?

Zeroth order in the derivative expansion

Recall:

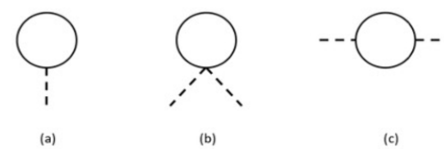
Renormalization of cosmological constant

$$\sqrt{-g}\Lambda = \Lambda(1 + \frac{1}{2}h^\sigma_\sigma + \frac{1}{8}(h^\sigma_\sigma)^2 - \frac{1}{4}h_{\sigma\lambda}h^{\sigma\lambda} + ...)$$

- example here due to massive scalar

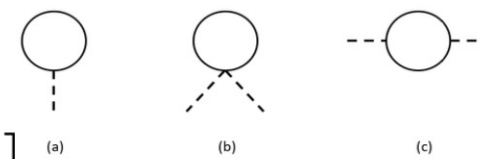
$$-i\mathcal{M} = i\frac{m^4}{32\pi^2}\left(\frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{\mu\nu}h_{\mu\nu}\right)\left[\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m^2} + \frac{3}{2}\right]$$

- no dependence on external momentum scales
- $\log\mu$ disappears along with $\frac{1}{\epsilon}$ upon renormalization



Zeroth order nonlocality

Leftover effects after renormalization are in two graviton amplitude



$$\mathcal{M}_{\mu\nu\alpha\beta} = \frac{1}{160\pi^2 q^4}\left(Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\alpha}\right)\left[m^4 J(q^2) + \frac{1}{6}m^2 q^2 - 3m^2 q^2 J(q^2)\right]$$

$$Q_{\mu\nu} = q_\mu q_\nu - \eta_{\mu\nu} q^2$$

$$J(q^2) = \int_0^1 dx\ \log\left[\frac{m^2 - x(1-x)q^2}{m^2}\right]$$

Logarithmic non-analyticity at energy scales above the mass

Note: The above is for a scalar particle

Fermion is -2 times this

Packaging as covariant effective action

First **as weak field – harmonic gauge**

$$\mathcal{L} = \frac{m^4}{160\pi^2}\left[h_{\mu\nu}\log[(\Box + m^2)/m^2]h^{\mu\nu}. - \frac{1}{8}h\log[(\Box + m^2)/m^2]h\right]$$

with $\langle x|\log[(\Box + m^2/m^2)|y\rangle = \int \frac{d^4q}{(2\pi)^4}e^{iq\cdot(x-y)}J(q^2)$

More generally match to **covariant form**

$$\begin{aligned}\langle RR\rangle &= 2Q_{\mu\nu}Q_{\alpha\beta} \\ \langle R_{\lambda\sigma}R^{\lambda\sigma}\rangle &= \frac{1}{4}\left[2Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\alpha}\right]\end{aligned}$$

leads to

$$\begin{aligned}\mathcal{L} &= \frac{m^4}{40\pi^2}\left[\left(\frac{1}{\Box}R_{\lambda\sigma}\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R^{\lambda\sigma}\right) - \frac{1}{8}\left(\frac{1}{\Box}R\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R\right)\right] \\ &+ \frac{m^2}{240\pi^2}\left[R_{\lambda\sigma}\frac{1}{\Box}R^{\lambda\sigma} - \frac{1}{8}R\frac{1}{\Box}R\right]\end{aligned}$$

Decoupling

This is only non-local above the particle's mass

$$J(q^2) \sim \log\frac{-q^2}{m^2} - 2 \qquad q^2 \gg m^2.$$
$$\sim -\frac{q^2}{6m^2} \qquad q^2 \ll m^2$$

This leads to a local Lagrangian of order 4 derivatives below mass scale

Finite range non-locality

Enormous numerical advantage over Λ

The renormalized value of Λ is extraordinarily small

- contributions of order m^4

This operator has ”normal” size, with $\frac{m_t^4}{\Lambda} \sim 10^{56}$ - not a free parameter

But in weak field limit, new operator starts at order h^2

Influence?

Use the harmonic gauge form – gets rid of pesky $\frac{1}{\nabla\hat{}2}$ terms

$$\mathcal{L} = \frac{m^4}{160\pi^2}\left[h_{\mu\nu}\log[(\Box + m^2)/m^2]h^{\mu\nu}. - \frac{1}{8}h\log[(\Box + m^2)/m^2]h\right]$$

Gauge condition

$$g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu} = 0$$

Flat FLRW type solution:

$$g_{\mu\nu} = \mathrm{diag}[a^3(t), -a(t), -a(t), -a(t)]$$

Approximation

Local approximation

$$a(t) = a_0\left[1 + \frac{\dot{a}}{a}(t-t_0) + \frac{1}{2}\frac{\ddot{a}}{a}(t-t_0)^2 + ...\right]$$

Cutoff approximation (decoupling)

$$\begin{aligned}\log\left(\frac{\Box + m^2}{m^2}\right) &= \log\frac{\Box}{m^2} \qquad t < \frac{1}{m} \\ &= 0 \qquad\qquad t > \frac{1}{m}\end{aligned}$$

Then integral can be evaluated - causal propagators (Frob, Roura, Verdaguer)

$$\int d^4x' \langle x|\log\frac{\Box}{m^2}|x'\rangle f(t') = \int dt' \lim_{\epsilon\to 0} 2\left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')(\log(m\epsilon)+\gamma)\right] f(t')$$

with the result

$$\int d^4x' \langle x|\log\frac{\Box + m^2}{m^2}|x'\rangle h_{\mu\nu}(t') \sim 2\frac{\dot{a}}{a}\frac{1}{m}\ \mathrm{diag}[3,-1,-1,-1]$$

Resulting local equation

Then the harmonic gauge FLRW equations are (for fermion loop)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho\, a^6 + \frac{2m^4}{5\pi^2}\left(\frac{\dot{a}}{a}\right)\frac{1}{m}\right]$$

$$\frac{\ddot{a}}{a} = 6\left(\frac{\dot{a}}{a}\right)^2 - \frac{4\pi G}{3}a^6(\rho + 3p)$$

Perturbative solution

$$a(t) = a_0(t) + \delta a(t)$$

$$\left(\frac{\delta \dot{a}}{a}\right) = \frac{4\pi G}{3}\left[\frac{2m^4}{5\pi^2}\frac{1}{m}\right]$$

$$\left(\frac{\dot{a}_0}{a}\right) \sim \sqrt{Gm^4} \qquad \frac{\delta \dot{a}}{a} \sim Gm^3$$

Only comparable for large m

Implications??

2b) Limits to effective field theory techniques

1) Obvious limit is at high energy and possibility of new DOF

$$\mathcal{M} = \mathcal{M}_0 \left[1 + Gq^2 + G^2 q^4 + ...\right]$$

2) Less obvious is extreme IR (maybe just a technical problem?)

- metric contains the dynamical field
- metric can get large even if curvature is everywhere small

$$g_{\mu\nu}(y) = \eta_{\mu\nu} + \frac{1}{3}R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}(y_0)y^\alpha y^\beta y^\gamma$$
$$+ \left[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta}(y_0) + \frac{2}{45}R_{\alpha\mu\beta\lambda}(y_0)R^\lambda{}_{\gamma\nu\delta}(y_0)\right] y^\alpha y^\beta y^\gamma y^\delta + \mathcal{O}(\partial^5)$$

- we do not have good tools to handle this
- also propagation past black holes…

3) Maybe also quantum mechanics of large systems

- limits to our experimental knowledge of QM – because of #2) gravity could be key

Summary

We have a quantum theory of general relativity

It is of the form of an effective field theory at ordinary energies
- not much different from low energy QED

There are reliable lessons at low energy

There are still more lessons to extract

Also – a new finite range nonlocal component of the effective action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